Extreme Value Statistics and Robust Filtering for Hydrological Data

WBS Herbstseminar 2014

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Wien, Nov. 06, 2014

The Need for Robustness in Extreme Value Theory:

An Illustrative Example

- want to estimate an extreme (here 99.5%) quantile

- ideal data: 1000 obs. from $\exp\left(\mathcal{N}(\mu = 3, \sigma = 2)\right)$
- true value in this example: 3470 *

- contamination: modify first 7 observations to $\sim 10^7$

- naïve estimation by empirical quantile:
  2960 (ideal), but 8910000 (contaminated)
- parametric (Max-Likelihood) estimation:
  3390 (ideal), but 7580 (contaminated)
- robust estimation (by rmx-procedure):
  3440 (ideal), but 3710 (contaminated)

*: all numbers rounded to **3** significant digits

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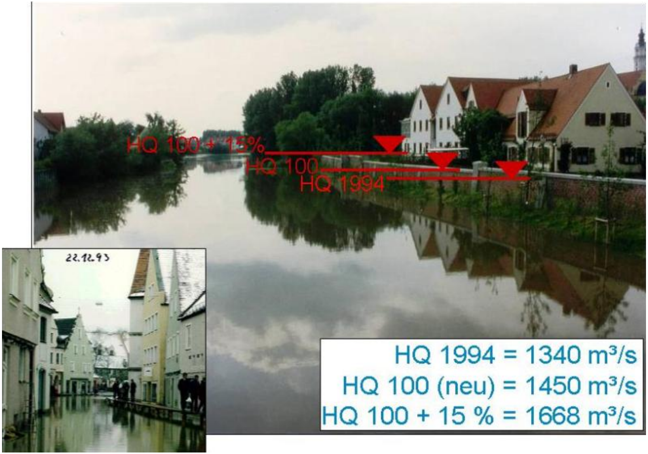
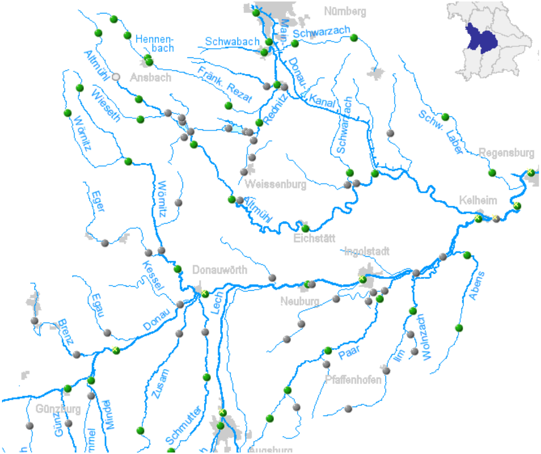
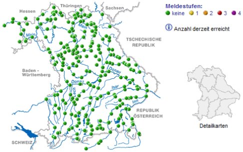
What are we talking about? — Floodings in Donauwoerth



source: `http://www.wwa-don.bayern.de/hochwasser/hochwasserschutzprojekte/donauwoerth`

Location of Donauwoerth in Bavaria





source: `http://www.hnd.bayern.de/`; traffic lights for alerts from Dec. 12, 2013

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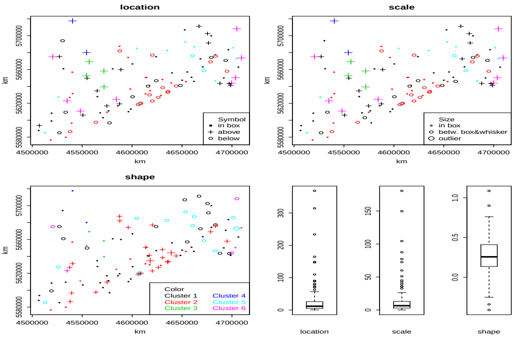
Current Approaches in Hydrology

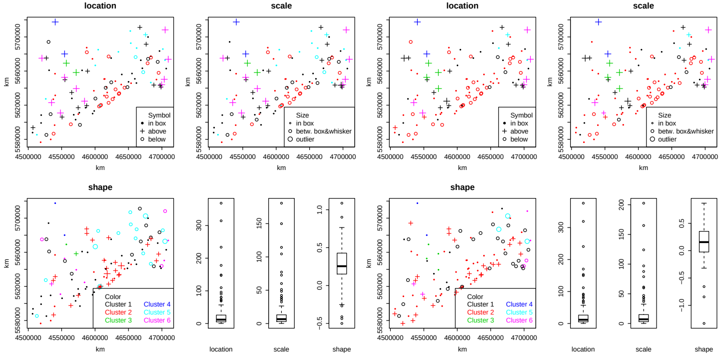
- techniques from **Extreme Value Statistics**
  see e.g. Reiß/Thomas[97], Katz et al.[02]
- geostastical aspects ⇝ **Regionalization** [borrowing strength]
  see e.g. Hosking/Wallis[97]



source: Laaha; clustering by GEV parameters (agnes) fit to block maxima in Saxony

Current Approaches in Hydrology (cont.)

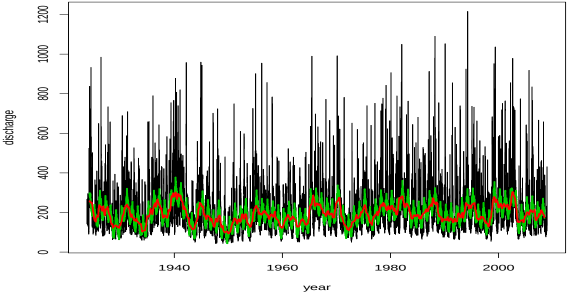
- robust estimation



clustering by ML vs. robust GEV parameters (agnes) fit to block maxima in Saxony

Issues

- trends and seasonalities:



in black: daily discharges; trend & seasonality; non-robust: c.f. Reiß and Thomas[07] /

robust: c.f. Fried et al.[07]

- outliers:
- dynamics:

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Issues

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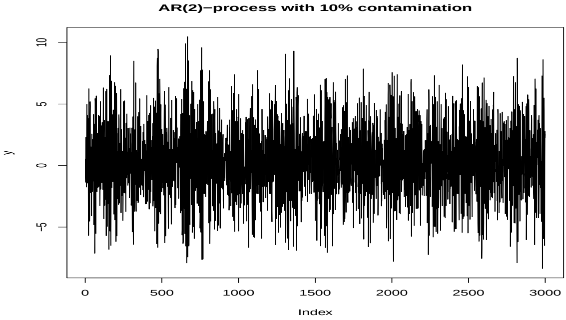
- outliers:

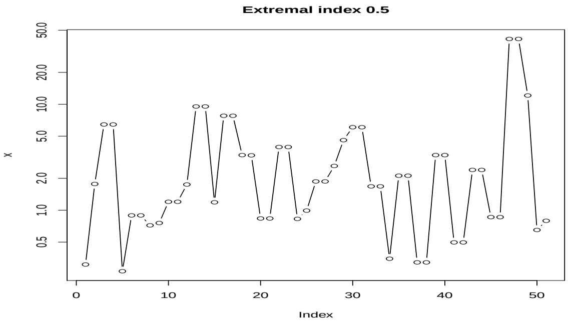


simulated data; often hard to distinguish between outliers and extremes

- dynamics:

Issues

- trends and seasonalities:
- outliers:
- dynamics:



toy data acc. to Coles[01], Ex. 5.1.

Research Questions and Challenges

- find models and **robust** procedures which
  - capture *extreme* behaviour
  - provide a simple & parsimonious, yet flexible *dynamics*
  - possibly account for *regional* effects

- address the question:

  inter-arrival time distribution of extremes

  no new question: see, e.g., Khaliq et al.[06]

State Space Models and Filtering Problem

Linear, Time–Discrete, Time-Invariant Euclidean Setup

ideal model:

$$
\begin{aligned}
x_t &= F x_{t-1} + v_t, & v_t &\overset{\text{indep.}}{\sim} (0, Q), \\
y_t &= Z x_t + \varepsilon_t, & \varepsilon_t &\overset{\text{indep.}}{\sim} (0, V), \\
& & x_0 &\sim (a_0, Q),
\end{aligned}
$$

$\{v_t\}, \{\varepsilon_t\}, x_0$ indep. as processes

(hyper–parameters Z known, F, Q, V to be estimated)

Generalizations also covered

- non-linear SSM's (by *Extended K.F.*, *Unscented K.F.*)
- time-varying F_t, Q_t, V_t depending on time-inv. param. θ

Algorithms for Filtering and Parameter Fitting

Filter/Smoothing Problem (for known hyp.-param.'s)

$\mathrm{E}\left|x_i - f_t(y_{1:j})\right|^2 = \min_{f_t}!,\quad$ with $y_{1:j} = (y_1,\ldots,y_j),\quad y_{1:0} := \emptyset$

class. solution: Kalman–Filter and –Smoother— Kalman[60]

optimal among linear [Gaussian setting: among all] filters & smoothers:

Initialization:	$x_{0\|0} = a_0$	
Prediction:	$x_{i\|i-1} = F x_{i-1\|i-1},$	$[\Delta x_i = x_i - x_{i\|i-1}]$
Correction:	$x_{i\|i} = x_{i\|i-1} + M_i^0 \Delta y_i,$	$[\Delta y_i = y_i - Z_i x_{i\|i-1}]$
Smoothing:	$x_{i\|T} = x_{i\|i} + J_i(x_{i+1\|T} - x_{i\|i}),$	$[J_i = \Sigma_{i\|i} F^\tau \Sigma_{i\|i-1}^{-1}]$

$+$ corresp. recursions for predict-/filter-/smoothing error cov.'s $\Sigma_{i|i[-1]}, \Sigma_{i|T}$ and Kalman gain M_i^0

route: Init. $\to$ "forward-loop" $= \{$Prediction, Correction$\}$ $\to$

$\to$ "backward-loop" $=$ Smoothing

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EM-Algorithm for SSM (unknown hyp.-param.'s)

application of EM-Algo to **SSMs** (with X_t as missings) by Shumway/Stoffer[82]; improvements by several authors since, see Durbin/Koopman[01].

Initialization: get initial estimators for F, Q, V, e.g. by moment-type-estimator

E–Step: reconstruct unobserved states by Kalman filter and smoother

M–Step: parameter estimation, e.g. by (conditional) ML estimator

route: Init. $\rightarrow$ "EM-loop" $=$ {E–Step, M–Step}

Elements of Extreme Value Statistics

two settings — consider

(a) *block maxima* or (b) *exceedances over some threshold*

(a) Fisher-Tippett-Gnedenko Theorem:

possible limit distributions of $\max(X_i)$ have

cdf $H_\theta(x) = \exp(-(1+\xi(x-\mu)/\beta)^{-1/\xi})$ (**GEVD** [= Gen. Extreme Value Distrib.])

(b) Pickands-Balkema-de Haan Theorem:

possible limit distr. of threshold exceedances have

cdf $F_\theta(x) = 1-(1+\xi(x-\mu)/\beta)^{-1/\xi}$ (**GPD** [= Gen. Pareto Distrib.])

- FTG-Thm $\Longleftrightarrow$ PBdH-Thm
- linked by same **Parameter** $\theta = (\xi, \beta, \mu)^\tau$:
  - shape ξ (≥ 0) (tail behavior)
  - scale β
  - location/threshold μ $(\leq x)$

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Interplay of SSM and EVT

- basic extreme value theorems cover i.i.d. situation

- in a dynamic, time-dependent setting:
  use concept of **extremal index**
  see Embrechts et al.[97]

  - non-parametric approach: is $\mathbf{1}/$limiting mean cluster size
  - compare Drees[03,08], Janßen/Drees[13], Janßen[10], ERCIM 13

here: dynamics captured by SSM
extremes modeled in the i.i.d. innovations

thus: flexible, parametric DGP to study inter-arrival times of exceedances *(—not only in the limit)*

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Outliers

- Outliers and extremes – a contradiction? Dell'Aquila/Embrechts[06]
- What makes an obs. an outlier? (—and not a regular extreme)
  - occur **rarely** (usually, **5%–10%**)
  - **u**ncontrollable, from **u**nknown distr. (may vary obs.-wise), **u**npredictable
  - have **no predictive power**
  - usually: no error-free separation from ideal obs.
- In dynamic setting

  exogenous outliers affecting only singular observations

$$\begin{array}{lcl} \text{AO} & :: & \varepsilon_t^{\text{re}} \sim (1-r_{\text{AO}})\mathcal{L}(\varepsilon_t^{\text{id}}) + r_{\text{AO}}\mathcal{L}(\varepsilon_t^{\text{di}}) \\ \text{SO} & :: & y_t^{\text{re}} \sim (1-r_{\text{SO}})\mathcal{L}(y_t^{\text{id}}) + r_{\text{SO}}\mathcal{L}(y_t^{\text{di}}) \end{array}$$

  endogenous outliers / structural changes

$$\text{IO} \quad :: \quad \xi_t^{\text{re}} \sim (1-r_{\text{IO}})\mathcal{L}(\xi_t^{\text{id}}) + r_{\text{IO}}\mathcal{L}(\xi_t^{\text{di}})$$

  but also trends, level shifts

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Robustness

component-wise robustification to tackle outlier issue

- Init-EM: use robust autocovariances, see Higham[02], S.[10]
- E-Step-EM: use *rLS-Filter* (Ruckdeschel[01,10]),

  i.e., in Corr., replace $M_i^0 \Delta y_i$ by $H_{b_i}(M_i^0 \Delta y_i)$, $H_b(x) = x \min\{1, b/|x|\}$

  use *rLS-Smoother* (Ruckdeschel, S., Pupashenko[14]),

  i.e., in Smooth., replace $J_i(x_{i+1|T} - x_{i|i})$ by $H_{\tilde{b}_i}(J_i(x_{i+1|T} - x_{i|i}))$
- M-Step-EM: use robust multiv. regression and scale est.'s

  (see Croux/Joossens[08], Agullo et al.[08], ERCIM 10)

  ...work in progress—soon some more on this ...
- EVT: use optimally-robust *RMXE*s to fit GPD, GEVD (Ruckdeschel/Horbenko [12,13]); extends and improves, a.o. Hosking et al.[85]

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Real Data Set

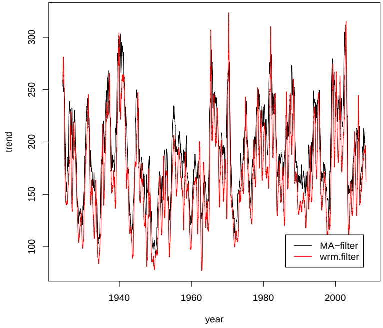
- daily average discharge data of *Danube river* in $[\mathrm{m}^3/\mathrm{s}]$
- location: gauge at *Donauwörth* (see initial pictures)
- start: 1923-11-01, end: 2008-12-31 ($> \mathbf{30,000}$ days)
- currently collected and provided by

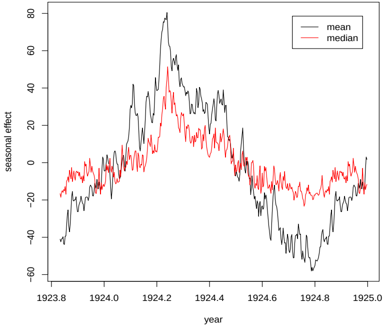
  Hochwassernachrichtendienst (HND),
  Bayerisches Landesamt für Umwelt (LfU)
  [translated: $\triangleq$ *Flooding news service by the Bavarian Environmental Office*]

- provided to us by G. Laaha within project *"Robust Risk Estimation"*

Model Specification

- the original data $y_t^\natural$ is first detrended and deseasonalized
  by moving averages to series y_t according to Reiß and Thomas[07] ,
  i.e., with *yearly trend* m_t and *yearly season* s_t, $y_t^\natural = m_t + s_t + y_t$
- robust trend/season extraction: c.f. Fried et al.[07]





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- for simplicity, assume an AR(4)-model for the y_t,
  —model order chosen by BIAR/IWLS (Martin/Thomson[82]) &
  ACM-type filter (Martin[79])
  i.e., in SSM context with the usual embedding

$$X_t = \begin{pmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \end{pmatrix}, F = \begin{pmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, Q = \begin{pmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, V = 0, Z^\tau = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

…entails that in M-Step-EM, we use standard robust MM regression lmrob

- to the tails of the obtained (filtered/smoothed) innovations $v_{t|T}$ fit a GPD model

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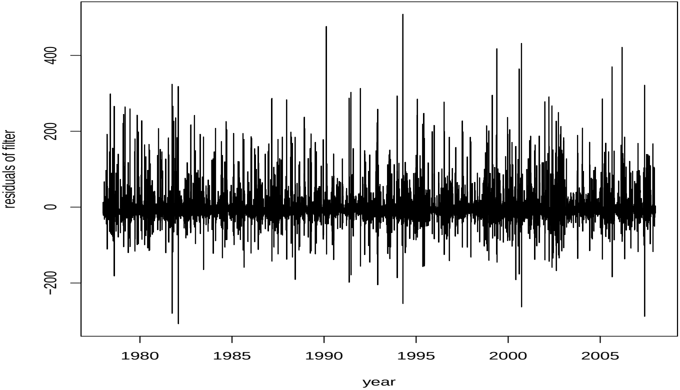
Effects of Filtering



detrended and deseasonalized y_t and observation residuals from rob. filter $\hat{\varepsilon}_t = y_t - Zx_{t|t}^{rob}$

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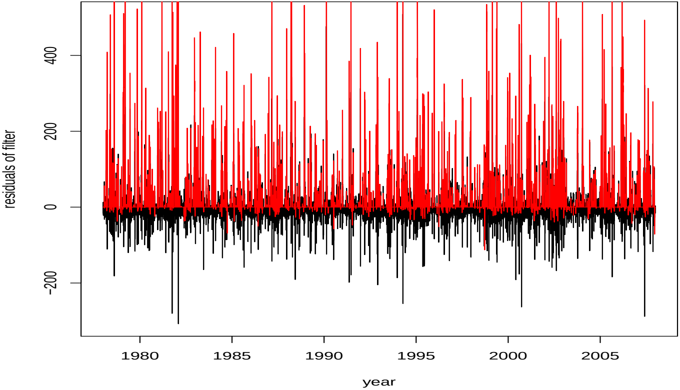

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Fit Real Data to GPD-Model

- raw data
  - threshold chosen with gpd.fitrange: 500

	scale	(SE)	shape	(SE)
MLE	125.68	(22.25)	-0.1766	(0.1245)
RMXE	123.49	(23.00)	-0.1289	(0.1810)

- with filtering
  - AR(4)-param's:

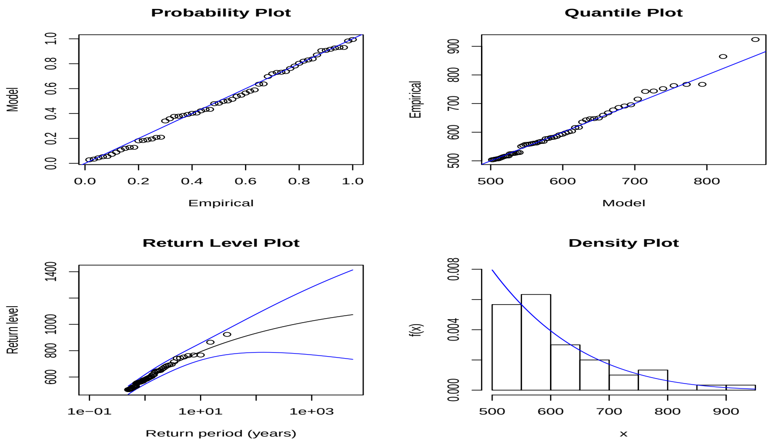
	φ_1	φ_2	φ_3	φ_4	μ	σ^2
MLE	1.2804	-0.5692	0.1825	-0.0044	0.1949	1721
rob	1.3078	-0.4492	0.1640	-0.0660	-15.6647	137

  - threshold chosen with gpd.fitrange: 250 (non-robust), 400 (robust)

	scale	(SE)	shape	(SE)
MLE	69.49	(17.62)	-0.0551	(0.1892)
RMXE	151.42	(19.94)	-0.0864	(0.1304)

for RMXE used functionality of R-pkgs ROptEst, RobExtremes

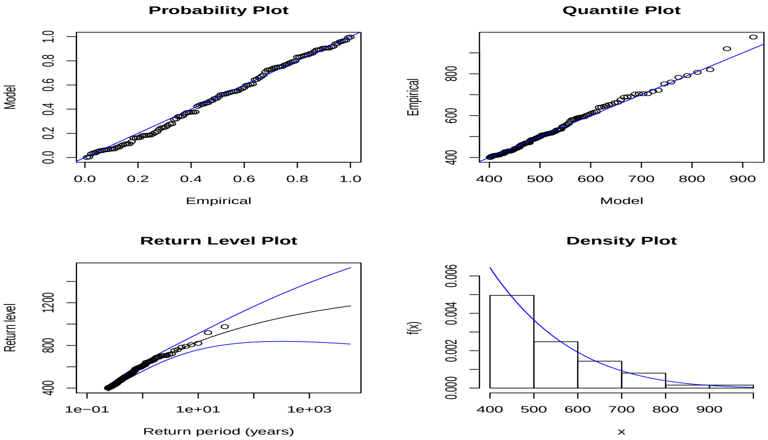
Tail and Fitted GPD on raw data



used

function gpd.**diag** of R-pkg `ismev`

Tail and Fitted GPD on filtered data



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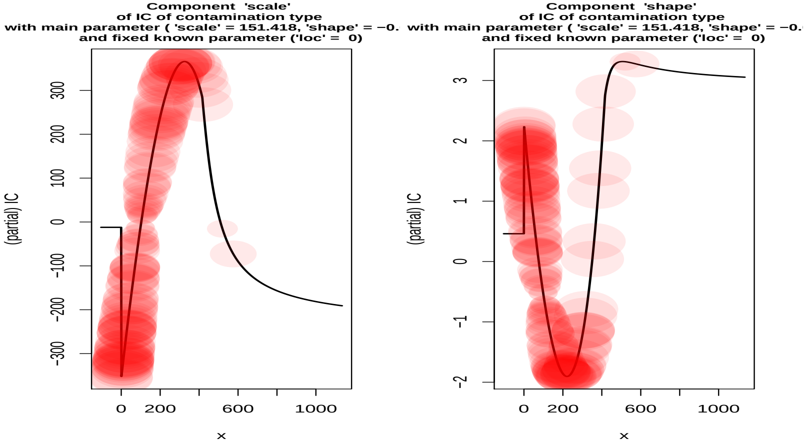
IC-Plot



used **plot**-method for `S4`-class `IC` of R-pkg `RobAStBase`

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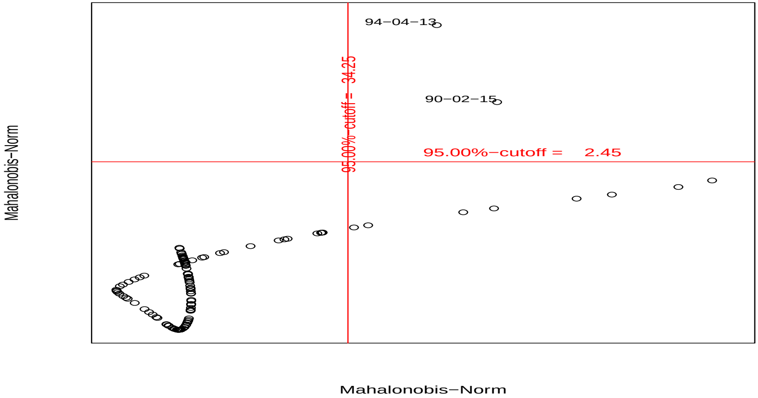
Outlyingness of Extremes



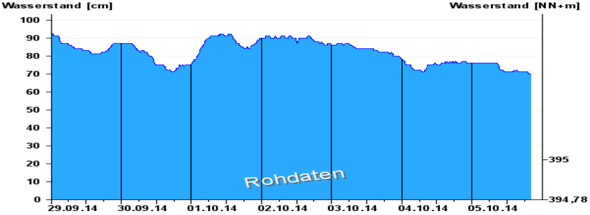
used function outlyingPlotIC of R-pkg RobAStBase

Outlyingness of Extremes II — True Effect?

Stammdaten | **Wasserstand** | Abfluss | Abflusstafel | Hochwassermarken | Mittel- / Höchstwerte
Gebietsdaten / Laufzeiten | Lagekarte / Bild | Jahrbuchseite

Darstellung in Tabellen-Form | Druckversion

Pegel im Donaugebiet: Donauwörth / Donau



Vorhersage: **keine** | 12-Std.-Vorhersage | 2-Tage-Trend
Linien: keine | Meldestufen | Hochwassermarken | **historische Ereignisse**

- Unsicherheitsbereich der Vorhersage(Erläuterung)
- Vorhersage vom **04.10.14 06:00** Uhr (Publikation: 13:02 Uhr)
- Letzter Messwert vom **05.10.14 19:45** Uhr: **70** cm

Zeitbereich auswählen:
Datum von: bis:

- **14.04.1994** Wasserstand: 577 cm
- **16.02.1990** Wasserstand: 553 cm
- **24.05.1999** Wasserstand: 552 cm
- **27.03.1988** Wasserstand: 544 cm
- **01.02.1982** Wasserstand: 543 cm

Conclusion

- presented a flexible, param. dynamic model class for hydrological extremes
- assessment of the inter-arrival distribution of extremes
- provided a step-by-step robustification
- evidence that robustification also enhances analysis of extremes

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THANK YOU FOR YOUR ATTENTION!

ROBUST RISK ESTIMATION

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