

*Vienna Section ROES*

# User-defined contrasts within multiple contrast tests- case studies using R

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# Differences Vienna- Hannover I

- *Weather, music, food, ...*
- sCI a MUST - see recent paper Phillips (2013):
  - 1 Because interpreting by decision makers, not statisticians
  - 2 Focusing on appropriate choice of **effect size** and their sCI
  - 3 (Adjusted) p-values are inappropriate from this perspective (although widely used)
  - 4 I.e. up to now: less focus on stepwise approaches or adaptive designs or gatekeeping (although 1991 ff papers)
- Nearly no FDR (just genome-wide Williams trend test using Benjamini-CI in package Isogene)
- Increasing power by:
  - 1 restricting the alternative
  - 2 taking the correlation(s) into account
  - 3 resulting in a **general non-product-moment structure**
  - 4 But after Ioannidis (2005) (Why most published research findings are false): **conservative is smart not painful- at least to some extend**

## Differences Vienna- Hannover II

- Using R only, i.e. in papers and packages
- Gaussian distribution only a possibility, focus on GLMM
- Less focus on weighted procedures (choice of weights?)
- Not just RCT, but also genetics, toxicology, molecular biology
- Triple: superiority, non-inferiority, equivalence (by means of sCI)
- Non-parametric as well (co-working with Goettingen group in a joint DfG-project)
- Robustness, e.g. variance heterogeneity

# The Problem I

- Charlies pioneering many21-procedure (Dunnett, 1955) is belonging to the most cited statistical papers. WebSci (04/2013): 4363 times
- Relevant up to now, e.g. for comparison of diversities in metagenomics (Pallmann et al., 2012)
- However, limited to **Gaussian errors with homogeneous variances**- and so in related software (SAS PROC MIXED, SPSS,...)
- But different endpoints occur commonly, e.g.:
  - i Proportions (Schaarschmidt and Biesheuvel, 2008)
  - ii Scores (count) data (Jaki et al., 2013)
  - iii Poly-3 estimates, i.e. mortality-adjusted tumor rates in carcinogenicity studies (Schaarschmidt et al., 2007)
  - iv Skewed-distributed endpoints:
    - a) *non-parametric (Konietschke and Hothorn, 2012),*
    - b) *log-normal (Schaarschmidt, 2013),*
  - v (Censored) time-to-event data (Herberich and Hothorn, 2012)

## The Problem II

- Even when Gaussian errors can be assumed, a diversity of problems exist:
  - i Inference for  $\mu_i/\mu_C$  instead of  $\mu_i - \mu_C$  (Dilba et al., 2004, 2007)
  - ii Correcting for heteroscedasticity in unbalanced designs (Hasler and Hothorn, 2008)
  - iii Multiple endpoints:
    - a) for superiority (Hasler and Hothorn, 2011),
    - b) for non-inferiority (Hasler and Hothorn, 2013)
  - iv Two-way layouts: claiming for or against qualitative interactions (Kitsche and Hothorn, 2013)
  - v Mixed models (Kruppa, 2009)
  - vi Using different contrasts, e.g. order restricted (Bretz, 2006), change-point (Hirotsu et al., 2011), vs. grand-mean (Djira and Hothorn, 2009)
  - vii Replacing global ANOVA F-test by MCT vs. grand mean (Konietschke et al., 2013)
- Focusing on **simultaneous confidence intervals (sCI)** (*instead of adj. p-values*): interpretability (*single step procedures so far*)

# MCP's formulated as MCT's I

- MCT: **multiple contrast test**
- A **contrast** is a suitable linear combination of estimators, e.g. means:

$$\sum_{i=0}^k c_i \bar{x}_i$$

- Here  $i = 0 \dots k$ , focusing on comparisons vs. control
- A **contrast test** is standardized

$$t_{\text{Contrast}} = \sum_{i=0}^k c_i \bar{x}_i / S \sqrt{\sum_{i=0}^k c_i^2 / n_i}$$

where  $\sum_{i=0}^k c_i = 0$  guaranteed a  $t_{df, 1-\alpha}$  distributed level- $\alpha$ -test

- To achieve compatible sCI  $\sum \text{sign}^+(c_i) = 1, \sum \text{sign}^-(c_i) = 1$
- Notice, arbitrary  $c_i$  can be used in resampling tests- **one reason for their popularity?**

## MCP's formulated as MCT's II

- A **multiple contrast test** is defined as maximum test:

$$t_{MCT} = \max(t_1, \dots, t_q)$$

which follows jointly  $(t_1, \dots, t_q)'$  a  $q$ -variate  $t$ -distribution with degree of freedom  $df$  and correlation matrix  $R$  ( $R = f(c_{ij}, n_i)$ )

- R-library(mvtnorm): (non)-central multivariate  $t$ -distribution for any correlation matrix (Mi et al., 2009; Genz et al., 2012) **r-,d-,q-,p-**
- One-sided lower **simultaneous confidence limits:**

$$[\sum_{i=0}^k c_i \bar{x}_i - S \cdot t_{q,df,R,1-sided,1-\alpha} \sqrt{\sum_i^k c_i^2 / n_i}]$$

# MCP's formulated as MCT's III

- The choice of a particular contrast matrix defines the MCT
- Known examples (balanced design  $k=2$  .... *just to keep it simple*)

Many-to-one, one-sided (Dunnett, 1955)

$c_i$	C	$T_1$	$T_2$
$c_a$	-1	0	1
$c_b$	-1	1	0

All pairs comparisons (Tukey 1953)

$c_i$	C	$T_1$	$T_2$
$c_a$	-1	0	1
$c_b$	-1	1	0
$c_c$	0	-1	1
$c_d$	1	-1	0
$c_e$	-1	1	0
$c_f$	0	1	-1

Change-point comparisons (Hirotsu et al., 2011)

$c_i$	C	$D_1$	$D_2$
$c_a$	-1	0.5	0.5
$c_b$	-0.5	-0.5	1

Williams-type procedure (Bretz, 2006)

$c_i$	C	$D_1$	$D_2$
$c_a$	-1	0	1
$c_b$	-1	1/2	1/2



# One- vs two-sided hypothesis I

- Simply using 2 different contrast matrices
- E.g. many21 One-sided:

$c_j$	C	$T_1$	$T_2$
$c_a$	-1	0	1
$c_b$	-1	1	0

Two-sided:

$c_j$	C	$T_1$	$T_2$
$c_a$	-1	0	1
$c_b$	-1	1	0
$c_c$	1	0	-1
$c_d$	1	-1	0

- Just two different related correlation matrices

## sCI for ratios of $\mu_j$ I

- **Aim:** simultaneous confidence intervals for  $\mu_i/\mu_0$

$$\omega_i = \mathbf{c}_i\boldsymbol{\mu}/\mathbf{d}_i\boldsymbol{\mu}$$

- $\mathbf{c}_i$  and  $\mathbf{d}_i$  are the  $i^{\text{th}}$  row vector of  $\mathbf{C}$  and  $\mathbf{D}$  for numerator and denominator
- E.g. for Dunnett-type contrasts

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

## sCI for ratios of $\mu_j$ II

- The simultaneous Fieller-type confidence intervals for  $\omega_j$  are the solutions of the inequalities

$$T^2(\omega_j) = \frac{L^2(\omega_j)}{S_{L(\omega_j)}^2} \leq t_{q,\nu,R(\omega),1-\alpha}^2,$$

with the numerator

$$L(\omega_j) = \sum c_i \bar{Y}_i - d_j \omega_j \bar{Y}_0,$$

Notice, Sasabuchi's trick of a linear form

- $t_{q,\nu,R(\omega_j),1-\alpha}$  is a central  $q$ -variate  $t$ -distribution with  $\nu$  degrees of freedom and correlation matrix  $R(\omega_j) = [\rho_{ij}]$ , where  $\rho_{ij}$  depend on  $c_{hi}$ ,  $n_i$  **and on unknown ratios**  $\omega_j$ : plug-in ML-estimators (Dilba et al., 2006) [Trick no. 2](#)
- The *mratios* R package (Dilba et al., 2007; Djira et al., 2012) can be used to make inferences about ratios of parameters in mixed models

## sCI when variance heterogeneity occurs I

- Variance heterogeneity is quite common, i.e.  $\varepsilon_{ij} \sim N(0, \sigma_i^2)$ .
- Standard MCP do not control FWER, particularly for unbalanced  $n_i$
- Modified test statistic  $T^{2*}(\omega_i) = L(\omega_i)^2 / S_{L(\omega_i)}^{2*}$ , where

$$S_{L(\omega_i)}^{2*} = \frac{\omega_i^2}{n_0} S_0^2 + \sum_{h=q+1-i}^q \frac{n_h}{\tilde{n}_i^2} S_h^2.$$

- $T^*(\omega_i)$  has an approximate  $t$ -distribution with approximate Satterthwaite-type  $\nu$   
Under variance heterogeneity: both  $\nu$  **and**  $R(\omega)$  depend on the **unknown ratios**  $\omega_i$  **and the unknown variances**  $\sigma_i^2$
- Plug-in modification: *sci.ratioVH* function in the R package *mratios* (Hasler and Hothorn, 2008)

# Non-parametric procedure I

- Commonly:  $H_0^F : F_0 = \dots = F_k$  formulated in terms of the distribution functions against simple tree  $H_1^F : F_0 < F_i$
- But the distribution of the rank means is unknown under  $H_1$ , neither sCI are available for a general unbalanced design, nor power can be estimated
- AND: tied or ordered categorical data, such as severity counts, should be analyzed as well
- AND: variance heterogeneity occurs frequently; therefore a Behrens-Fisher (BF) version is needed

## Non-parametric procedure II

- Using relative effect size (Brunner and Munzel, 2000), (Ryu and Agresti, 2008):

$$p_{01} = \int F_0 dF_1 = P(X_{01} < X_{11}) + 0.5P(X_{01} = X_{11}).$$

- $p_{01}$  is a *win probability* in the sense of Hayter (2013)
- $p_{01}$  can be interpreted for trials with subjects Browne (2010)
- **sCI**: Konietzschke (2011) Let  $R_{sj}^{(0s)}$  denote the rank of  $X_{sj}$  among all  $n_0 + n_s$  observations within the samples 0 and s
- The rank means can be used to estimate  $p_{0s}$

$$\hat{p}_{0s} = \frac{1}{n_0} \left( \bar{R}_{s\cdot}^{(0s)} - \frac{n_s + 1}{2} \right)$$

## Non-parametric procedure III

- Asymptotically  $\sqrt{N}(\hat{\rho}_1 - \rho_1, \dots, \hat{\rho}_q - \rho_q)'$  follows a central multivariate normal distribution with expectation  $\mathbf{0}$  and covariance matrix  $\mathbf{V}_N$  (Konietschke, 2011)
- Related approximate  $(1 - \alpha)100\%$  one-sided lower simultaneous confidence limits are:

$$\left[ \hat{\rho}_\ell - t_{q, \nu, \mathbf{R}, 1-\alpha} \sqrt{\mathbf{S}_\ell} \right], \ell = 1, \dots, q, \quad (1)$$

- E.g. relative Shirley-type effects for order restriction (Shirley, 1977)

$$\begin{aligned} \rho_1 &= \rho_{0k} \\ \rho_2 &= \frac{n_{k-1}}{n_{k-1} + n_k} \rho_{0(k-1)} + \frac{n_k}{n_{k-1} + n_k} \rho_{0k} \\ &\vdots \\ \rho_q &= \frac{n_1}{n_1 + \dots + n_k} \rho_{01} + \dots + \frac{n_k}{n_1 + \dots + n_k} \rho_{0k} \end{aligned}$$

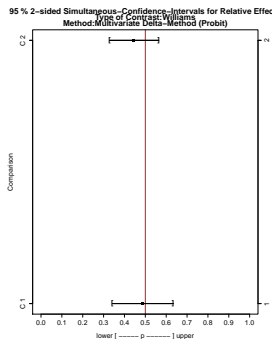
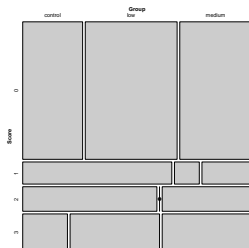
# Non-parametric procedure IV

- Shirley-type test for graded histopathological findings using R package *nparcomp*

Ordered categorical findings of non-neoplastic lesions in the P-Cresidine carcinogenicity study: hyperplasia in parotid gland

```
library(nparcomp)
```

```
nparcomp(Score~Group, data=parotid, asy.method = "probit", type="Willi
```





# sCI for proportions I

- Three approaches
  - 1 *Wald-type (Hothorn et al., 2008)*
  - 2 *Add1- adjusted (Schaarschmidt and Biesheuvel, 2008)*
  - 3 *Profile likelihood (Gerhard, 2010)*
- For sample sizes of  $n_i = 50 \dots 10$  there is no hope for valid  $(1 - \alpha)100\%$  Wald intervals. Therefore we need confidence intervals with coverage probability approximately 95% also for smaller samples
- And, for almost all proportions a **one-sided** alternative for an increase/decrease is appropriate
- As effect size the difference of proportions is common (alternatively RR, OR)

## sCI for proportions II

- One-sided, lower  $(1 - \alpha)100\%$  Wald-type confidence limits for the difference of the proportions of treatments against C:

$$\left[ \sum_{i=1}^I c_i p_i - z_{q,R,1-\alpha} \sqrt{\sum_{i=1}^I c_i^2 \hat{V}(p_i)}; \right]$$

with  $\hat{V}(p_i) = p_i(1 - p_i)/n_i$  and  $z_{q,R,1-\alpha}$  denoting the  $(1 - \alpha)$  quantile of the  $q$ -variate normal distribution

- **$R$**  depends not only on the known contrast coefficients  $c_{im}$  and sample sizes  $n_i$  **but also on the unknown  $\pi_j$  and  $V(p_j)$**  where the plug-in of the ML-estimators  $\hat{\pi}_j$  and  $\hat{V}(\pi_j)$  works well.

## sCI for proportions III

- Agresti and Coull (1998) showed that adding a total of four pseudo-observations to the observed successes and failures yields approximate confidence intervals for one binomial proportion with good small sample performance
- One-sided limits were investigated by Cai (2005) in the case of a single binomial proportion

$$\left[ \sum_{i=1}^I c_i \tilde{p}_i - z_{q,R,1-\alpha} \sqrt{\sum_{i=1}^I c_i^2 \tilde{V}(\tilde{p}_i)} \right]$$

- Choice of simultaneous confidence limits

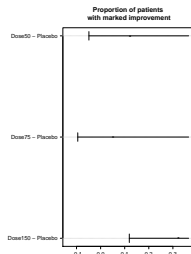
Notation	$\tilde{p}_i$	$\tilde{V}(p_i)$
Wald	$Y_i/n_i$	$p_i(1-p_i)/n_i$
add-1	$(Y_i + 0.5)/(n_i + 1)$	$\tilde{p}_i(1-\tilde{p}_i)/(n_i + 1)$
add-2	$(Y_i + 1)/(n_i + 2)$	$\tilde{p}_i(1-\tilde{p}_i)/(n_i + 2)$

## sCI for proportions IV

- Simulation study (Schaarschmidt et al., 2008): use add1 approx. one-sided lower limits when  $n_i$  not too small
- Example: Simultaneous confidence limits for tubular epithelia hyaline droplet degeneration in male rats by means of MCPAN.

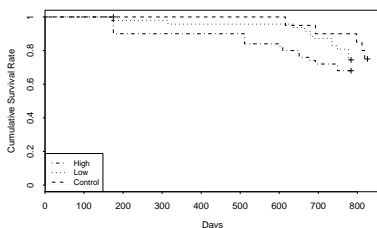
	Control	Dose50	Dose75	Dose150
with degeneration	2	6	4	13
n	32	27	32	21

```
library(MCPAN)
data(liarozole)
plot(binomRDci(tab, type="Dunnett", alternative="greater", method="ADD1"))
```



# sCI for time-to-event data I

- Williams-type proc. comparing survival functions: i) Cox proport. hazards model or ii) the frailty Cox model to allow a joint analysis over sex and strains (Herberich and Hothorn, 2012)
- Example: Mortality in NTP-TR120 carcinogenicity



- Effect size: Hazard rate. Using Williams-type sCI

Comparison	Estimated HR	sim. 97.5%-Interval
C vs. D2	3.83	[0.82, $\infty$ )
C vs. (D1, D2)	3.18	[0.71, $\infty$ )

# A Dunnett-type approach for multiple endpoints I

- In RCT with several primary correlated endpoints **and** a multi-arm design, **multiplicity adjustment should take both the endpoints and the treatment comparisons** into account, i.e. global control of FWER
- Extension of the Dunnett procedure (Dunnett, 1955) for  **$k$  multiple endpoints and  $q$  comparisons**

$$\{X_{ipj} : p = 1, \dots, k\} \sim \perp N_k(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}) \quad (i = 0, \dots, q, j = 1, \dots, n_i).$$

- I.e. unknown covariance matrices  $\boldsymbol{\Sigma}_i \in \mathbb{R}^{k \times k}$  with possibly different variances and covariances for the endpoints, but the same covariance matrices for all treatments
- Testing the hypotheses

$$H_0^{(ip)} : \eta_{ip} \leq \delta_p$$

## A Dunnett-type approach for multiple endpoints II

- This is a **union-intersection-test** because the overall null hypothesis of interest can be expressed as an intersection of the local null hypotheses, i.e.,

$$H_0 = \bigcap_{i=1}^q \left\{ \bigcap_{p=1}^k H_0^{(ip)} \right\}.$$

- This means that the overall null hypothesis  $H_0$  is rejected if and only if a local null hypothesis  $H_0^{(ip)}$  is rejected for **at least one treatment for at least one endpoint**.
- The test of the above hypotheses based on (now for the difference!)

$$T_{ip} = \frac{\bar{X}_{ip} - \bar{X}_{0p} - \delta_p}{S_p \sqrt{\frac{1}{n_i} + \frac{1}{n_0}}} \quad (i = 1, \dots, q, p = 1, \dots, k).$$

## A Dunnett-type approach for multiple endpoints III

- The distribution of the univariate  $\mathbf{T}_i$  under  $H_0^{(i)}$  is simply a  $k$ -variate  $t$ -distribution with  $\nu$  degrees of freedom and the correlation matrix  $\mathbf{R}$ , i.e.,

$$\mathbf{T}_i \sim t_{k,\nu,\mathbf{R},1-\alpha}.$$

- Consequently, under  $H_0$ , the vector of **all** test statistics,

$$\mathbf{T} = (\mathbf{T}'_1, \dots, \mathbf{T}'_q)' = (T_{11}, \dots, T_{ip}, \dots, T_{qk})',$$

follows approximately a  $qk$ -variate  $t$ -distribution with  $\nu$  degrees of freedom and a correlation matrix denoted by  $\tilde{\mathbf{R}}$ , i.e.,

$$\mathbf{T} \stackrel{\text{appr.}}{\sim} t_{qk,\nu,\tilde{\mathbf{R}},1-\alpha}$$



## A Dunnett-type approach for multiple endpoints IV

The correlation matrix  $\tilde{\mathbf{R}}$  is given by

$$\tilde{\mathbf{R}} = (\mathbf{R}_{ii'})_{i,ii'} = \begin{pmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \cdots & \mathbf{R}_{1q} \\ \mathbf{R}_{12} & \mathbf{R}_{22} & \cdots & \mathbf{R}_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{1q} & \mathbf{R}_{2q} & \cdots & \mathbf{R}_{qq} \end{pmatrix}.$$

The submatrices  $\mathbf{R}_{ii'} = (\rho_{ii',pp'})$  describe the correlations between the  $i$ th and the  $i'$ th comparison for all endpoints. Their elements are

$$\rho_{ii',pp'} = \begin{cases} \rho_{pp'}, & i = i' \\ \rho_{pp'} \frac{1}{\sqrt{\left(\frac{n_0}{n_i} + 1\right) \left(\frac{n_0}{n_{i'}} + 1\right)}}, & i \neq i' \end{cases}$$

# A Dunnett-type approach for multiple endpoints V

- **sCI** The lower limits of the approximate  $(1 - \alpha)100\%$  sCIs for  $(\eta_{11}, \dots, \eta_{qk})'$  are given by

$$\hat{\eta}_{ip}^{low} = \bar{X}_{ip} - \bar{X}_{0p} - t_{qk, \nu, \hat{\mathbf{R}}, 1-\alpha} S_p \sqrt{\frac{1}{n_i} + \frac{1}{n_0}}$$

- The R package SimComp was developed

# Some user-defined contrasts I

**I: Concept: claim-wise error rate** Phillips (2013)

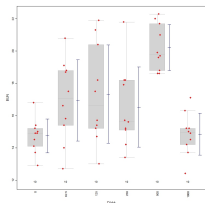
**II: Regulatory toxicology**

- US-NTP recommends the use of Dunnett and Williams procedure. Which one really? Take both! (Jaki and Hothorn, 2013)

<i>Dun</i>					<i>Wil</i>					<i>UWil</i>				
<i>c<sub>qi</sub></i>	<i>NC</i>	<i>D<sub>1</sub></i>	<i>D<sub>2</sub></i>	<i>D<sub>3</sub></i>	<i>c<sub>qi</sub></i>	<i>NC</i>	<i>D<sub>1</sub></i>	<i>D<sub>2</sub></i>	<i>D<sub>3</sub></i>	<i>c<sub>qi</sub></i>	<i>NC</i>	<i>D<sub>1</sub></i>	<i>D<sub>2</sub></i>	<i>D<sub>3</sub></i>
<i>c<sub>a</sub></i>	-1	0	0	1	<i>c<sub>a</sub></i>	-1	0	0	1	<i>c<sub>a</sub></i>	-1	0	0	1
<i>c<sub>b</sub></i>	-1	0	1	0	<i>c<sub>b</sub></i>	-1	0	1/2	1/2	<i>c<sub>b</sub></i>	-1	0	1/2	1/2
<i>c<sub>c</sub></i>	-1	1	0	0	<i>c<sub>c</sub></i>	-1	1/3	1/3	1/3	<i>c<sub>c</sub></i>	-1	1/3	1/3	1/3
										<i>c<sub>d</sub></i>	-1	0	1	0
										<i>c<sub>e</sub></i>	-1	1/2	1/2	0
										<i>c<sub>f</sub></i>	-1	1	0	0

## Some user-defined contrasts II

- Blood urea nitrogen content after 13 weeks repeated administration of sodium dichromate dihydrate on male rats (NTP2012)



Comparison	Dun	Wil	DuWi	UWil
1000 - 0	0.80	0.60	0.80	0.80
500 - 0	6.8e-07	-	8.1e-07	8.4e-07
250 - 0	0.110	-	0.11	0.12
125 - 0	0.017	-	0.018	0.020
62.5 - 0	0.045	-	0.047	0.051
(1000 + 500)/2 - 0	-	0.0013	0.0030	0.0033
(1000 + 500 + 250)/3 - 0	-	0.0029	0.0057	0.0063
(1000 + 500 + 250 + 125)/4 - 0	-	0.0021	0.0037	0.0042
(1000 + 500 + 250 + 125 + 62.5)/5 - 0	-	0.0022	0.0039	0.0043
(500 + 250)/2 - 0	-	-	-	< 0.001
(500 + 250 + 125)/3 - 0	-	-	-	< 0.001
(500 + 250 + 125 + 62.5)/4 - 0	-	-	-	< 0.001
(250 + 125)/2 - 0	-	-	-	0.023
(250 + 125 + 62.5)/3 - 0	-	-	-	0.015
(125 + 62.5)/2 - 0	-	-	-	0.015

## Some user-defined contrasts III

### III: Genetic association studies

Association between a di-allelic marker and a disease can be presented in a  $2 \times 3$  contingency table, where  $aa$  is the high risk candidate allele and  $AA$  is any of the other alleles

	$aa$	$aA$	$AA$	Total
Cases	$r_{aa}$	$r_{aA}$	$r_{AA}$	$r$
Controls	$s_{aa}$	$s_{aA}$	$s_{AA}$	$s$
Total	$n_{aa}$	$n_{aA}$	$n_{AA}$	$n$

The global null hypothesis for the unknown proportions  $\pi_j = E(r_j/n_j), j \in \{aa, aA, AA\}$

$$H_0 : \pi_{aa} = \pi_{aA} = \pi_{AA}$$

can be compared to either a global heterogeneity alternative

$$H_1^{\text{heterogeneity}} : \pi_j \neq \pi_{j'}, j \neq j' \in (aa, aA, AA)$$

e.g. by Pearson  $\chi^2$  test

# Some user-defined contrasts IV

or to a global order restricted alternative

$$H_1^{\text{ordered}} : \pi_{aa} \leq \pi_{aA} \leq \pi_{AA}$$

$H_1^{\text{ordered}}$  can be decomposed in three elementary alternatives

$$H_1^{\text{additiv}} : \pi_{aa} < \pi_{aA} < \pi_{AA}$$

$$H_1^{\text{dominant}} : \pi_{aa} < \pi_{aA} = \pi_{AA}$$

$$H_1^{\text{recessive}} : \pi_{aa} = \pi_{aA} < \pi_{AA}$$

# Some user-defined contrasts V

- 1 The quadratic form of the global heterogeneity test can be replaced by MCT against grand mean (Konietschke et al., 2013)

	$\pi_{aa}$	$\pi_{aA}$	$\pi_{AA}$
any wild type vs. risk	-1	0.5	0.5
heterozygotes vs. homocyg.	0.5	-1	0.5
any risk allele vs. homocyg. wild type	0.5	0.5	-1

- 2 For the order restricted alternatives the contrast coefficients  $c_{jq}$  are:

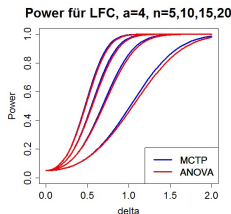
	$\pi_{aa}$	$\pi_{aA}$	$\pi_{AA}$
additive	-1	0	1
dominant	-1	0.5	0.5
recessive	-0.5	-0.5	1

- 3 Together:

	$\pi_{aa}$	$\pi_{aA}$	$\pi_{AA}$
additive mode	-1	0	1
dominant mode= any homocygotes vs GM	-1	0.5	0.5
recessive mode=risk homocygotes vs GM	-0.5	-0.5	1
over-dominance mode =heterocyg. vs homocyg	0.5	-1	0.5

# Replacing ANOVA F-test by MCT vs. grand mean I

- Analyzing one-way layouts by F-test or Kruskal-Wallis test is common
- Quadratic F-test can be replaced by **max-test of linear contrasts vs. grand mean** Konietschke et al. (2013)



- Power:
  - similar for least favorable configuration,
  - larger or smaller for any alternatives
- sCI available



# R libraries - LUH and friends I

- multcomp
- mvtnorm
- mratio
- MCPAN
- SimComp
- goric
- mcprofile
- AND: pairwiseCI, BSagri, simboot, PropCIs, binMto

# Take Home Message I

- (Single step) **sCI** are available for **most endpoints, designs and contrast formulations**
- Related R packages are available: UseR!
- (Not shown: power for the compatible tests can be calculated under some assumptions)
- I.e. unified analysis of all end-points in a trial/study is possible
- Alternative: hypothesis-restricted AIC-based model selection, e.g. for MED estimation (Kuiper et al., 2013)
- Focus now: mixed model applications

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